

CRITICAL POINT IN A 3-3-1 MODEL WITH RIGHT-HANDED NEUTRINOS

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Abstract

The boson mass spectrum of a 3-3-1 model with right-handed neutrinos is investigated by tuning a unique free parameter within the exact algebraical approach for solving gauge models with high symmetries. A very strange coincidence of the masses in both the neutral and the charged boson sectors can occur at a not very high breaking scale. This could explain why the masses of the new bosons have not been exactly weighted in the laboratories by date. In order to have good phenomenological behaviour in the neutrino sector, one can resort to either the conservation of the global lepton number $L_e - L_\mu - L_\tau$ or see-saw mechanism.

PACS numbers: 12.60.Cn; 14.70.Pw.

Key words: 3-3-1 models, boson mass spectrum.

1 Introduction

In this brief report we would like to emphasize the fact that in the 3-3-1 gauge model with right-handed neutrinos, a critical point could well occur at a not very high breaking scale. All the Particle Data [1] suggest that the neutral boson Z' of any extension of the Standard Model (SM) has to be heavier than neutral boson of the SM. However, the case when the "old" neutral boson screens the "new" one - namely, when their masses are identical - seems to be ignored. A very suitable manner to investigate this issue is supplied by the exact algebraical approach for solving gauge theories with high symmetries proposed several years ago by Cotăescu [2] and developed by the author in a series of recent papers [3] - [7] on the 3-3-1 model with right-handed neutrinos. For some interesting details of the phenomenology in such models, the reader is referred to Refs. [8] - [35].

The results regarding the exact boson mass spectrum - presented hereafter - are amazing and they look plausible from phenomenological viewpoint. There is a critical point in the model where all the neutral bosons of the theory Z , Z' and Y gain the same mass! Moreover, at the same time the charged bosons W^\pm and X^\pm are indistinguishable on the mass reason too. This special feature seems to explain why the new bosons were not yet exactly "weighted" in laboratory. Only their lower energy bounds are suggested in the literature (see on this issue Ref. [1] and references therein).

The paper is organized as follows. Section 2 briefly reviews the main results of the theoretical method employed to solve the particular 3-3-1 model with right-handed neutrinos, so the boson mass spectrum - depending on the sole free parameter a - is given. Section 3 analyses the circumstances under which the critical point occurs and estimates the phenomenological implications of the particular value of the parameter a that ensures this behaviour. Last section is devoted to our conclusions.

2 Boson Mass Spectrum

The particle content of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$ gauge model with right-handed neutrinos, under consideration here [8] - [35], is:

Lepton families

$$f_{\alpha L} = \begin{pmatrix} \nu_\alpha^c \\ \nu_\alpha \\ e_\alpha \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, -1/3) \quad (e_{\alpha L})^c \sim (\mathbf{1}, \mathbf{1}, -1) \quad (1)$$

Quark families

$$Q_{iL} = \begin{pmatrix} D_i \\ -d_i \\ u_i \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 0) \quad Q_{3L} = \begin{pmatrix} T \\ t \\ b \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, -1/3) \quad (2)$$

$$(b_L)^c, (d_{iL})^c \sim (\mathbf{3}, \mathbf{1}, -1/3) \quad (t_L)^c, (u_{iL})^c \sim (\mathbf{3}, \mathbf{1}, +2/3) \quad (3)$$

$$(T_L)^c \sim (\mathbf{3}, \mathbf{1}, +2/3) \quad (D_{iL})^c \sim (\mathbf{3}, \mathbf{1}, -1/3) \quad (4)$$

with $i = 1, 2$. The numbers in paranthesis denote - in a self-explanatory notation - the representations and the charecters with respect to each group involved in the theory.

With these representations this particular 3-3-1 model stands anomaly free, as one can easily check out by using little algebra. Note that, although all the anomalies cancel by an interplay between families, each family still remains anomalous by itself.

These representations can be achieved starting with the general method [2] of exactly solving gauge models with high symmetries by just chosing an appropriate set of parameters (for certain details of dealing with the algebraical procedure and the special parametrisation involved here, the reader is referred to Ref. [7]). They are:

$$e, \theta_W, \nu_0 = 0, \nu_1 = 0, \nu_2 = 1 \quad (5)$$

established by experimental arguments (e, θ_W) [1] or by internal reasons of the general method (ν_i) [2].

Along with the above parameters, one must add some new ones - as they determine the Higgs sector of the model - grouped in a parameter matrix which reads:

$$\eta^2 = (1 - \eta_0^2) \text{Diag} \left[1 - a, \frac{1}{2}(a + b), \frac{1}{2}(a - b) \right] \quad (6)$$

where, for the moment, a and b are arbitrary non-vanishing real parameters that ensure the condition $\text{Tr}(\eta^2) = (1 - \eta_0^2)$. At the same time, $\eta_0, a \in [0, 1)$.

They will determine, after the spontaneous symmetry breakdown (SSB) - which takes place up to the universal residual $U(1)_{em}$ one - a non-degenerate boson mass. The exact expressions of the boson masses are given by the Eqs. (53) - (55) in Ref. [2], namely

$$M_i^j = \frac{1}{2} g \langle \phi \rangle \sqrt{[(\eta^{(i)})^2 + (\eta^{(j)})^2]} \quad (7)$$

for the non-diagonal gauge bosons which usually are charged but, as one can easily observe in the 3-3-1 model under consideration here, one of them must be neutral, and

$$(M^2)_{ij} = \langle \phi \rangle^2 \text{Tr}(B_i B_j) \quad (8)$$

with

$$B_i = g \left[D_i + \nu_i (D\nu) \frac{1 - \cos \theta}{\cos \theta} \right] \eta \quad (9)$$

for the diagonal bosons of the model. The angle θ is the rotation angle around the versor ν orthogonal to the electromagnetic direction in the parameter space [2]. The versor condition holds $\nu_i \nu^i = 1$.

Since the electro-weak sector of the model is described now by the chiral gauge group $SU(3)_L \otimes U(1)_Y$, the two diagonal generators D_1 and D_2 (D s - stands for the Hermitian diagonal generators of the Cartan subalgebra) in the fundamental representation of $SU(3)_L$ are: $D_1 = T_3$ and $D_2 = T_8$ - connected to the Gell-Mann matrices in the manner $T_a = \lambda_a/2$ - and $D_0 = I$ for the chiral hypercharge.

In the 3-3-1 model under consideration here, the relation between θ in the general method [2] and the Weinberg angle θ_W from SM was established [3, 7] and it is

$$\sin \theta = \frac{2}{\sqrt{3}} \sin \theta_W \quad (10)$$

Boson mass spectrum Using Eq. (7) one can express the masses of the non-diagonal bosons. They are (according to the parameter order in matrix η^2):

$$m_W^2 = m^2 a \quad (11)$$

$$m_X^2 = m^2 \left[1 - \frac{1}{2}(a + b) \right] \quad (12)$$

$$m_Y^2 = m^2 \left[1 - \frac{1}{2}(a - b) \right] \quad (13)$$

Throughout this paper we consider $m^2 = g^2 \langle \phi \rangle^2 (1 - \eta_0^2)/4$.

Evidently, W is the "old" charged boson of the SM which links positions 2 - 3 in the fermion triplet, namely the left-handed neutrino to its charged lepton partner and, respectively, the "up" quarks to "down" quarks. The neutral Y boson couples the left-handed neutrino to the right-handed one, and the "classical" up (down) quarks to the "exotic" up (down) quarks, that is positions 1 - 2 in fermion triplet are involved. The remaining X boson is responsible for the charged current between positions 1 - 3 in each triplet.

The "pure" neutral bosons (diagonal ones) acquire mass by diagonalizing the resulting matrix:

$$M^2 = m^2 \begin{pmatrix} 1 - \frac{1}{2}a + \frac{1}{2}b & -\frac{1}{\sqrt{3-4s^2}} \left(1 - \frac{3}{2}a - \frac{1}{2}b\right) \\ -\frac{1}{\sqrt{3-4s^2}} \left(1 - \frac{3}{2}a - \frac{1}{2}b\right) & \frac{1}{3-4s^2} \left(1 + \frac{3}{2}a - \frac{3}{2}b\right) \end{pmatrix} \quad (14)$$

after combining Eqs. (8), (9) and (6), where the notation $\sin \theta_W = s$ has been made for simplicity.

One of the two diagonal bosons has to be identical to the neutral boson Z from SM. Therefore, the latter should be an eigenvector of this mass matrix corresponding to the eigenvalue $m_Z^2 = m_W^2 / \cos^2 \theta_W$ firmly established in the SM [1].

$$M^2 |Z\rangle = \frac{m^2 a}{1-s^2} |Z\rangle \quad (15)$$

That is, one computes $\text{Det} [M^2 - m^2 a / (1-s^2)] = 0$ which leads to the constraint upon the parameters $b = a \tan^2 \theta_W$. Consequently, the parameter matrix (6) becomes:

$$\eta^2 = (1 - \eta_0^2) \text{diag} \left[1 - a, \frac{a}{2 \cos^2 \theta_W}, \frac{a}{2} (1 - \tan^2 \theta_W) \right] \quad (16)$$

Under these circumstances, the boson mass spectrum yields:

$$m_W^2 = m^2 a \quad (17)$$

$$m_X^2 = m^2 \left(1 - \frac{a}{2 \cos^2 \theta_W} \right) \quad (18)$$

$$m_Y^2 = m^2 \left[1 - \frac{a}{2} (1 - \tan^2 \theta_W) \right] \quad (19)$$

$$m_Z^2 = \frac{m^2 a}{\cos^2 \theta_W} \quad (20)$$

$$m_{Z'}^2 = m^2 \left[1 + \frac{1}{3 - 4 \sin^2 \theta_W} - a \left(1 + \frac{\tan^2 \theta_W}{3 - 4 \sin^2 \theta_W} \right) \right] \quad (21)$$

since $\text{Tr}(M^2) = m_Z^2 + m_{Z'}^2$, holds.

We obtained a mass spectrum depending on the free parameter a (to be tuned). One can observe that, although the fermion representations and even the order in the parameter matrix η^2 are not the same with those chosen in Ref. [3], the resulting mass spectrum has the same structure. That means there are equivalent ways to choose the parameters in the general method in order to reach the same particle content of the model and the same physics.

3 Critical Point

When inspecting the boson mass spectrum - Eqs. (17) - (21) - one can enforce certain conditions on the parameter a as to obtain realistic values, in accordance with the available experimental data. Furthermore, the neutrino phenomenology was investigated [4] and, because a very high breaking scale $\langle\phi\rangle$ was required, the method led to a natural see-saw mechanism [5]. It was thus implemented in order to keep consistency of the tiny masses of neutrinos (and their specific mixing angles) with the reasonable masses of the new bosons. Mass values in the region of TeVs [1] were allowed for the new bosons, since this see-saw mechanism was exploited by inserting a supplemental small parameter in the η^2 matrix.

However, a special and unexplored yet opportunity is offered by our method. As long as the exact masses of the new bosons have not been experimentally determined to date, one is entitled to ask if there is no screening between them. Namely, if the new neutral boson does not "cover" the old one. More specifically, if their masses do not coincide? What kind of consequences has such a hypothesis?

From Eqs. (20) and (21) results that the free parameter has to be

$$a = \frac{2 \cos^2 \theta_W}{3 - 2 \sin^2 \theta_W} \quad (22)$$

in order to achieve $m_Z = m_{Z'}$. That is $a \simeq 0.6$ if we consider $\sin^2 \theta_W \simeq 0.223$.

Furthermore, what are the values gained by the masses of the remaining bosons? Embedding (22) in (17), (18) and (19) respectively, one obtains the amazing results:

$$m_W^2 = m_X^2 = m^2 \frac{2 \cos^2 \theta_W}{3 - 2 \sin^2 \theta_W} \quad (23)$$

and

$$m_Z^2 = m_{Z'}^2 = m_Y^2 = m^2 \frac{2}{3 - 2 \sin^2 \theta_W} \quad (24)$$

. These are the well-known values predicted by SM, namely 91.2 GeV for the neutral bosons, and 80.4 GeV for the charged ones. Assuming that in the SM

$$m_W = \frac{g}{2} \langle\phi\rangle_{SM} \quad (25)$$

holds, one can estimate the required breaking scale $\langle\phi\rangle$ of the 3-3-1 model under consideration here, by comparing it to (17). That is $\langle\phi\rangle \geq \langle\phi\rangle_{SM} / \sqrt{a}$. This leads to $\langle\phi\rangle \geq 320 \text{ GeV}$.

4 Concluding remarks

We have proven in this brief report that the exact algebraical approach for solving gauge models with high symmetries offers - when it is applied to a 3-3-1 model with right-handed neutrinos - a plausible explanation for the undiscovered yet new bosons predicted by such a model. They could well be "screened" by the "old" bosons, since a particular critical value is assigned to the free parameter a of the method. This can occur at a not very high breaking scale $\langle\phi\rangle \geq 320\text{GeV}$, while the whole SM content is naturally recovered (as it was shown in Ref. [7]).

Now, if one wants to deal with the correct neutrino phenomenology, one has to adjust the above solution, since such a value for the free parameter - $a \simeq 0.6$ - leads to an unacceptable order of magnitude for the individual masses in the neutrino sector where some tensor-like products between Higgs triplets are employed in the generating mass terms [3, 4]. There are several ways out of this unnatural outcome in order to accommodate the small neutrino masses in this model: (a) one can resort to the well-known see-saw mechanism (as in Ref. [5]) by adding a small new parameter in the matrix η^2 without spoiling these results, or (b) one can impose certain supplemental global symmetries (like, for instance, $L_e - L_\mu - L_\tau$) [6] where also a $\mu - \tau$ interchange symmetry can be invoked, or (c) one can even conceive a suitable radiative mechanism like in Refs. [13, 25, 33] that gives rise to the neutrino masses only at one-loop or two-loop level.

Therefore, we consider that the strange coincidence that simultaneously occurs - namely, $m_W^2 = m_X^2$ and $m_Z^2 = m_{Z'}^2 = m_Y^2$ - for a particular value of the free parameter seems more than a simple "fit". It seems to express a possible deeper identity between the "same charge" bosons. This hypothesis can not be ruled out *a priori*, since more accurate results regarding the decays of the "new" bosons and high-energy scatterings involving their couplings to fermions - experimental details which can reveal some restrictions on the parameter a - have to be more exactly investigated at LHC.

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